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Structural modal excitation using travelling impulses—force frequency shifting

L.L. Koss*

Department of Mechanical Engineering, Monash University, Clayton, Victoria 3800, Australia

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Abstract

A review of existing hardware and methods for vibration testing of large structures is given by Koss and has shown that the size of inertial vibration shakers, to achieve a specific displacement, has to increase, as a structure becomes larger. In previous papers the concept of "force frequency shifting (ffs) for structural excitation", was introduced to develop a more compact structural vibration exciter than is presently available for low frequencies. An ffs shaker operates at a frequency much greater than the natural frequency of the structure under test but generates a modal force at the lower frequency of the structure. This effect is accomplished by moving a vibrating force back and forth across the structure while the force is applied normally to its surface. For example, the generalized force generated by an ffs shaker at the fundamental structural frequency for a simply supported beam is given by $1.65\,P\,r/l$ where P is the high frequency out of balance force, r is the throw amplitude and l is the beam length. The term that reduces the efficiency of force transfer from high to low frequencies is "r/l" as, usually, the length of a structure is much greater than the throw of the force. This paper introduces another force frequency shifting approach that allows r/l to be large. This is accomplished by placing force exciters along a structure–spatial array, spaced a distance ΔX apart, and each force exciter is activated for a short period of time to simulate a travelling force traversing the structure forwards and backwards. The "force throw r "can thus be made large. Results of simulations and experiments verify that force frequency shifting can be accomplished using travelling impulses and modal identification can be achieved.
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1. Introduction

The concept of "force frequency shifting" has been given in previous work by Koss [1–3], and Trethewey and Koss [4–6] to develop a vibratory force shaker that is smaller, less weight, than

*Tel.: +61-3-9905-4000; fax: +61-3-9905-1825.

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existing inertial shaker for excitation of large structures. A review of existing methods of exciting structures and objectives of vibration testing are given by Koss [7], some of these objectives and techniques are given here. Integrity of structures such as bridges can be determined by measuring modal properties [8], other studies are employed to develop vibration control procedures [9] and other investigations are employed to determine modal properties [10]. As structural size increases the size of inertial force exciters increases to achieve a given response [7]. The objective of the study of force frequency shifting (ffs) vibration shakers is the development of hardware to achieve lighter weight low-frequency vibration exciters. The present concept requires moving the high-frequency shaking exciter, also, at a high frequency back and forth along a structure. A large oscillating force is required to accomplish this task and makes the concept impractical thus another approach to shift force frequency was investigated and is described in this paper; this concept will now be reviewed.

An ffs shaker generates a force that is normal to the surface of a structure under test at a high frequency f_1, Hz, and is moved back and forth along the structure at frequency f_2, Hz; shown in Fig. 1 is a schematic of the ffs technique. This action generates a dynamic moment that acts on the structure that contains a difference frequency, $f_1 - f_2$, and a sum frequency, $f_1 + f_2$. The difference frequency is the low frequency that can be employed for excitation of structures with low modal frequencies. A generalized (low frequency) modal force, Q_1, that is developed by an ffs shaker is dependent on the type of structure that is excited, e.g., cantilever beam, and has the following general format:

$$Q_1/P = cr/l, \tag{1}$$

where P is the force generated at the high frequency, c is a structure dependent constant, e.g., 1.65 for a simply supported beam and 5.0 for a free–free beam, see Ref. [8], r is the back and forwards throw of the vibration force and l is the structural length. The ratio r/l reduces the efficiency of force transfer from a high frequency to a low frequency. The force P is the out of balance force and is given by

$$P = m_{ob}e(2\pi f_1)^2, \tag{2}$$

where m_{ob} is the imbalance mass, e is the imbalance mass eccentricity and f_1 is the high frequency in Hz. To make force frequency shifting worthwhile f_1 has to be sufficiently high to reduce the

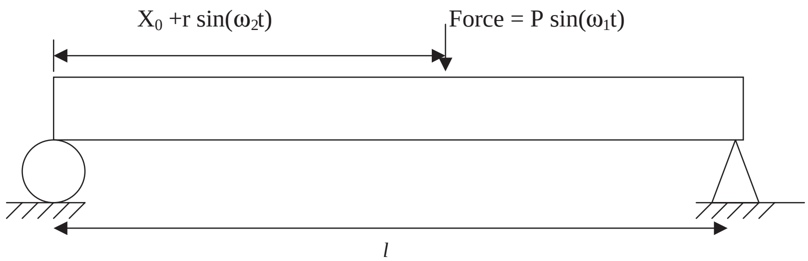

Fig. 1. Schematic of force frequency shifting principle shown. Simply supported bridge of length l, force amplitude of P, frequency of force is ω_1, amplitude of oscillation of r, equilibrium position of force of x_0, and oscillation frequency of ω_2. $\omega_1 = 2\pi f_1$ and $\omega_2 = 2\pi f_2$.

effect of the ratio r/l on the force shifting. Another approach to shifting a force efficiently is to increase r/l by have several force exciters positioned on a structure some distance apart and each exciter generates a portion of a high-frequency sine wave force to simulate a travelling load by turning "on and off" in a sequential manner. Thus, the term "r" can be increased greatly (up to half the length of the structure) and the frequency f_1 will also a high value. Previous work by Koss et al. [11] using a finite element routine demonstrated the feasibility of this approach. This paper examines and reports on this concept using simulations and experimental results.

2. Theory of travelling impulsive forces and simulations

The generalized force, Q_i, for a set of impulsive forces or moments that act on a flexible structure is given [12] as

$$Q_i = \int \{f(x,t)\Phi_i(x) + M(x,t)\Phi_i'\}\,\mathrm{d}x, \tag{3}$$

where $f(x,t)$ is a distributed force acting on a structure, $M(x,t)$ is a distributed moment acting on a structure, $\Phi_i(x)$ is the ith mode shape for the structure and $\Phi_i'(x)$ is the mode shape slope and x is position along the structure. The force term, $f(x,t)$, may be due to a set of force exciters acting on a structure located a distance Δx apart, or similarly for moment exciters. Thus, a continuous distribution of force can be replaced by a set of force exciters that act along a structure. Each exciter is actuated sequentially in time such that the force appears to be moving along the structure; such a force can be represented by

$$\begin{aligned} F(x,t) = F_0 \sin(2\pi f_1 t)\{&\delta(x-\Delta x-x_0)\delta(t-\delta t) \\ &+\delta(x-2\Delta x-x_0)\delta(t-2\Delta t)+\delta(x-3\Delta x-x_0)\delta(t-3\Delta t)+\cdots\}. \end{aligned} \tag{4}$$

The force amplitude is F_0, the force frequency is f_1 in Hz, Δx is the distance between force exciters, x_0 is the starting position for the force exciters on the structure, Δt is the time taken for the force to travel from one force exciter to the next and δ is the dirac delta function. The force exciters can be sequenced such that the force excitation appears to travel forward and backward along the structure. Eq. (4) is substituted into Eq. (3) for the calculation of the generalized force Q_1 for the first mode of vibration. The frequency at which the force moves forwards and backwards along the structure is given by f_2 in Hz. An example of a force sequence that acts along a 1 m long cantilever beam is shown in Fig. 2, where a 1 N force amplitude is employed, f_1 is 11.5 Hz, f_2 is 10 Hz, x_0 is 120 mm and Δx is 90 mm. The force exciters are sequenced in the order 1–3–5–7–8–6–4–2–1–3 etc., to simulate a forward and backward force movement along the structure. The calculations for generalized force were simulated in Matlab for 8 force exciters

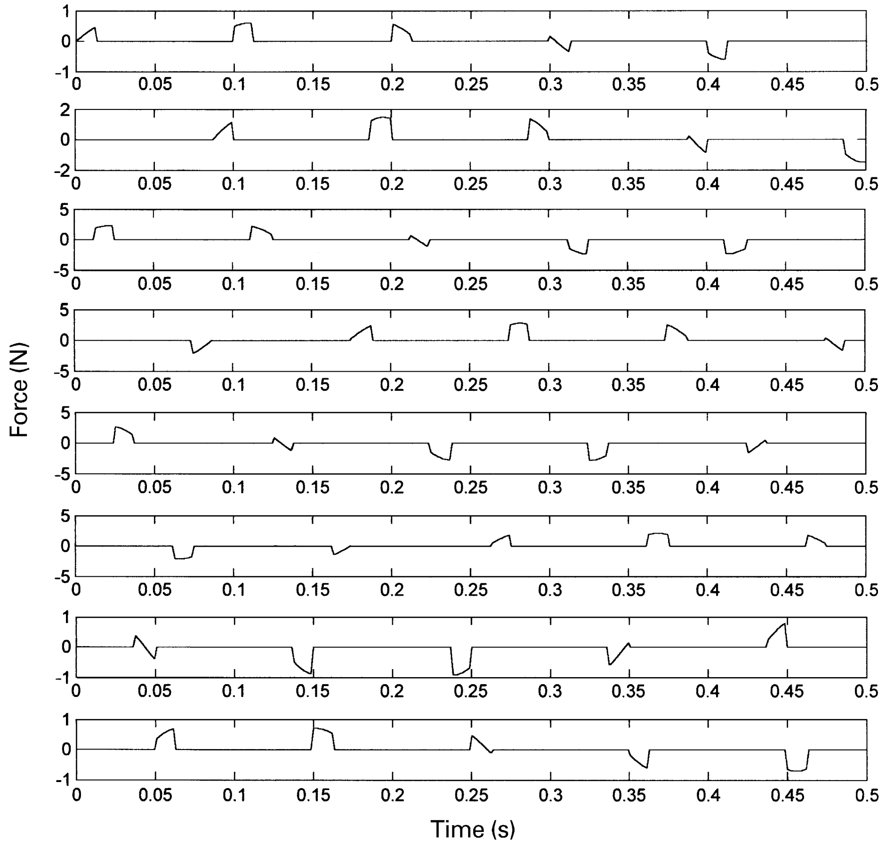

Fig. 2. Time histories of generalized force at each of the force exciter positions on a cantilever beam. Top trace is force of exciter at 120 mm from fixed end, no.1; lowest trace is force at 750 mm from fixed end, no. 8. Time sequence of forces is 1–3–5–7–8–6–4–2–1–3 etc. Thus, the force travels forwards and backwards along the beam.

using the following code:

$$\begin{gathered}
\text{f1} = \text{Force.} * (1 + \text{square}(2 * \text{pi} * \text{fr2} * \text{t}, 12.5)) * \text{thim}(1, 1);\\
\text{f2} = \text{Force.} * (1 + \text{square}(2 * \text{pi} * \text{fr2} * \text{t} - 1.75 * \text{pi}, 12.5)) * \text{thim}(2, 1);\\
\text{f3} = \text{Force.} * (1 + \text{square}(2 * \text{pi} * \text{fr2} * \text{t} - \text{pi}/4, 12.5)) * \text{thim}(3, 1);\\
\text{f4} = \text{Force.} * (1 + \text{square}(2 * \text{pi} * \text{fr2} * \text{t} - 1.5 * \text{pi}, 12.5)) * \text{thim}(4, 1);\\
\text{f5} = \text{Force.} * (1 + \text{square}(2 * \text{pi} * \text{fr2} * \text{t} - \text{pi}/2, 12.5)) * \text{thim}(5, 1);\\
\text{f6} = \text{Force.} * (1 + \text{square}(2 * \text{pi} * \text{fr2} * \text{t} - 5 * \text{pi}/4, 12.5)) * \text{thim}(6, 1);\\
\text{f7} = \text{Force.} * (1 + \text{square}(2 * \text{pi} * \text{fr2} * \text{t} - 3 * \text{pi}/4, 12.5)) * \text{thim}(7, 1);\\
\text{f8} = \text{Force.} * (1 + \text{square}(2 * \text{pi} * \text{fr2} * \text{t} - \text{pi}, 12.5)) * \text{thim}(8, 1);
\end{gathered} \tag{5}$$

where force is given by $F_0 \sin(2\pi f_1 t)/2$, F_0 is 1 N, the first natural frequency of the beam is 1.50 Hz and thim(j, 1) is the mode shape value for force exciter position j and mode number 1 pi is π, and

$fr2$ is the traversing frequency f_2. The square wave function with 8 different phases from 0° to $1.75\,\pi$ rad and an "on time" of 12.5% for each exciter were employed to give the results shown in Fig. 2.

The forces given in Eq. (5) were then employed on a modal basis to simulate the beam response, using Matlab's ODE 45 Runga–Kutta integrator with constant time step, on a modal basis, i.e., one mode at a time as follows, q_i is the generalized co-ordinate for mode i,

$$M_i\,\mathrm{d}^2q_i/\mathrm{d}t^2 + C_i\,\mathrm{d}q_i/\mathrm{d}t + K_iq_i = \text{Force as given in Eq. (5).} \tag{6}$$

The terms M_i, C_i and K_i are the modal mass, damping and stiffness for mode i.

2.1. Simulated response of a cantilever beam—travelling forces

The response of a cantilever beam, whose properties are given in Table 1, was simulated using Eq. (5) and the Matlab ODE 45 integration routine and eight excitation points; this beam was tested in the laboratory using eight PZTexciters. Results of the simulations are given as time histories and Fourier spectra of the modal response at a position on the beam at which the mode shape has a value of 1, comparisons of responses for mode two when excited with a difference frequency equal to the first natural frequency, the difference frequency equal to the second natural frequency and the effects of two, four and eight force exciters on response amplitude of the beam. The simulations used a 1 N force amplitude and the response is given for a 25 s analysis period; this duration allows the response to build up to a steady state.

A 25 s time history response of mode 1 for f_1 equal to 11.518 Hz and f_2 equal to 10 Hz is shown in Fig. 3 and the corresponding frequency spectrum is shown in Fig. 4; the difference frequency clearly excites the first mode. The use of f_1 equal to 5.518 Hz and f_2 equal to 4 Hz as forcing frequencies gives the same frequency spectrum as shown in Fig. 4; the amplitude results are the same as shown in Table 2. This analysis demonstrates that the same response is obtained as long as the force amplitude F_0 is the same and the difference frequency is the same. The response of mode 2 to the same force employed for mode 1 i.e., F_0 of 1 N, f_1 is 11.5 Hz and f_2 is 10 Hz, is given as a frequency spectrum in Fig. 5. The second mode at 9.51 Hz is poorly stimulated in comparison to being excited by frequencies of f_1 equal to 19.51 Hz and f_2 equal to 10 Hz as shown by the frequency spectrum of Fig. 6; the spectral line located at 0.48 Hz is due to a difference frequency between 19.51 and 2×10 Hz. Thus, the travelling impulse technique can be used to select, or excite, a given mode of vibration.

Table 1
Properties of 0.91 m long cantilevered steel beam of width of 35 mm and depth of 1.5 mm

	Mode 1	Mode 2	Mode 3
Natural frequency, Hz	1.52	9.51	26.64
Modal mass, kg	0.35	0.35	0.35
Modal stiffness, N/m	31.7	1250	9600
Modal damping ratio	0.01	0.01	0.01

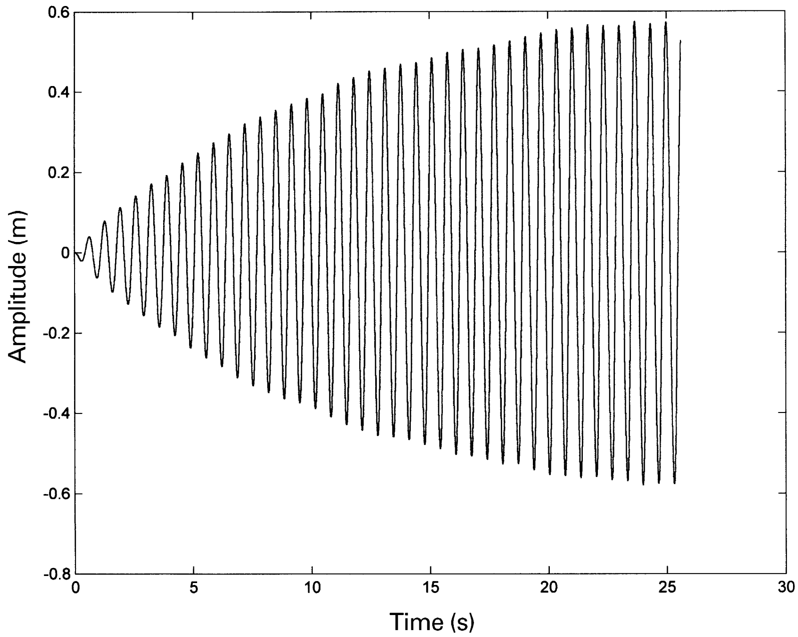

Fig. 3. Time history of mode 1 response (generalized co-ordinate) for forcing frequency of 11.518 Hz and oscillation frequency of 10 Hz; eight exciters are being used.

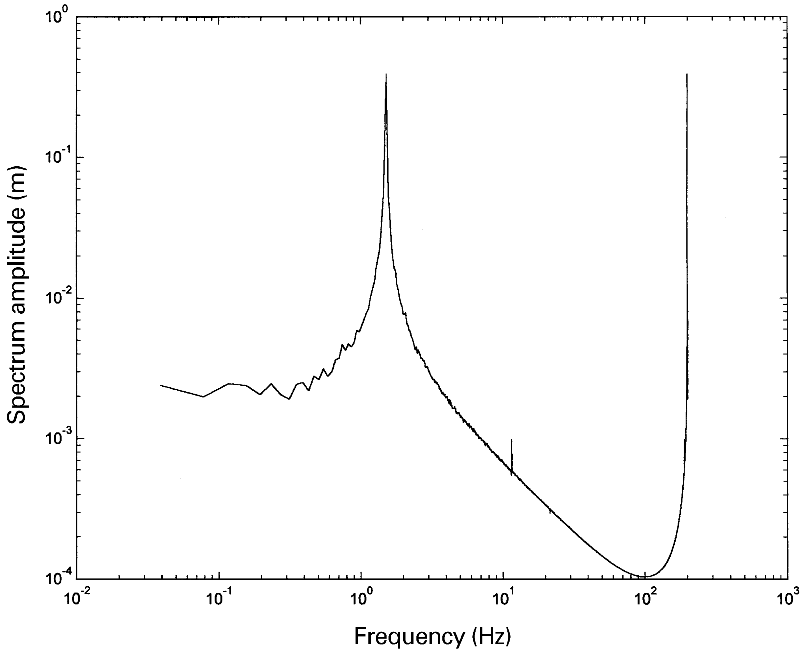

Fig. 4. Frequency spectrum of mode 1 response shown in Fig. 3.

Table 2
Simulated vibration mode 1 response due to different combinations of f_1 and f_2

f_1 (Hz)	f_2 (Hz)	R.m.s. (m) over 25 s
5.52	4	0.45
11.52	10	0.45

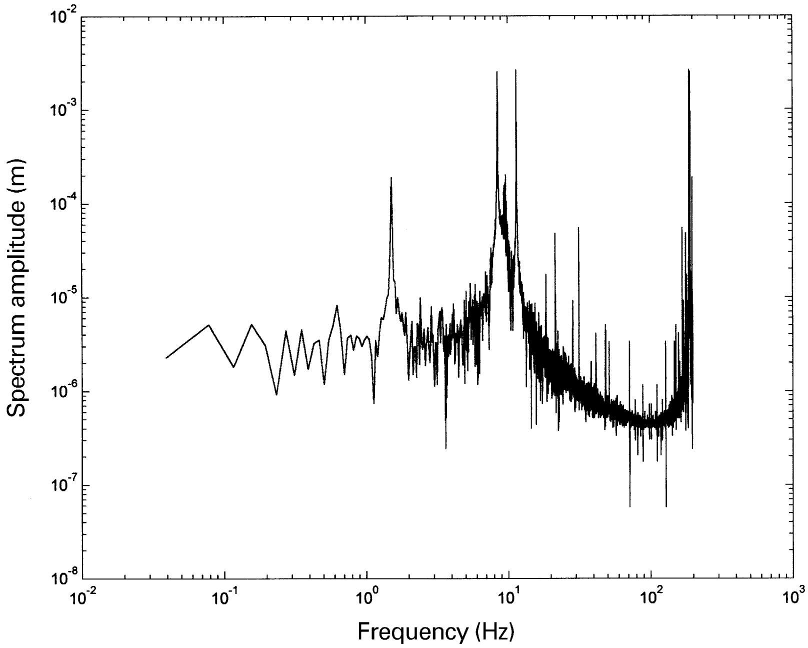

Fig. 5. Mode 2 response frequency spectrum for forcing frequency of 11.518 Hz and oscillation frequency of 10 Hz.

2.2. Travelling moment excitation of a cantilever beam

The force term, F_0, in Eq. (4) can be replaced by a moment term, M_0, to simulate a moment that travels along the structure and the mode shape slopes replace the mode shapes. Shown in Fig. 7 is the result of simulation of the first mode using point moments rather than forces; vibration mode 1 is readily excited, however, higher frequencies are present of almost equal amplitude to that at 1.518 Hz. The root mean square response of mode 2 due to excitation of several different combinations of f_1 and f_2 is given in Table 3; the results are similar although the frequency combinations are different.

2.3. Effects of number of exciters

The effects of the number of exciters on modal response for a cantilever beam were determined by simulation. A set of response results using two, four and eight force-exciters is given in Table 4.

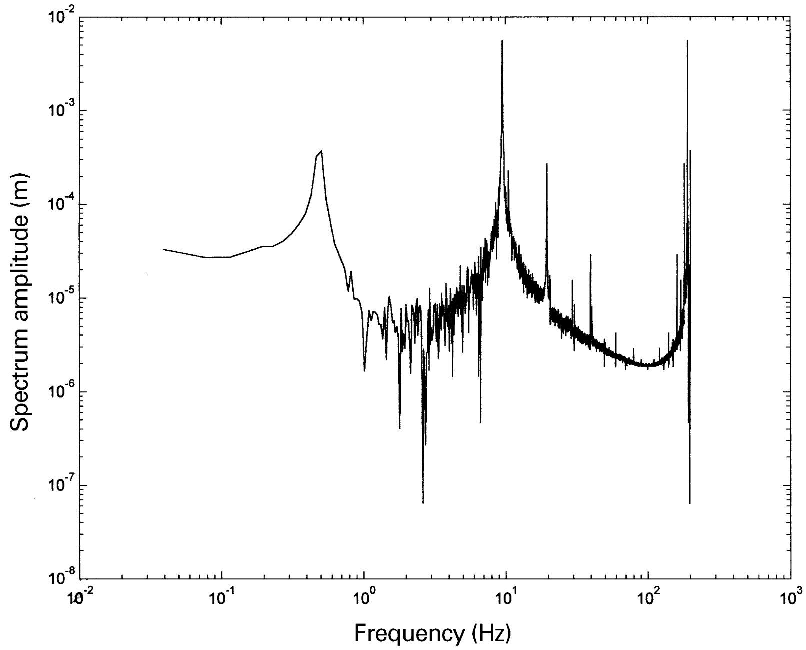

Fig. 6. Frequency spectrum of mode 2 response, natural frequency of 9.51 Hz, for forcing frequency of 19.51 Hz and oscillation frequency of 10 Hz; eight exciters are being used.

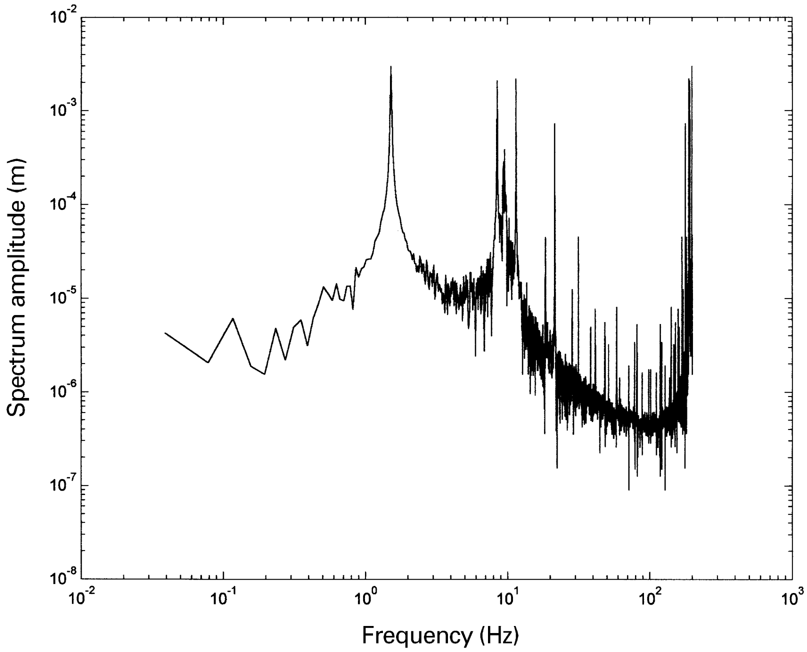

Fig. 7. Frequency spectrum of mode 1 response for forcing frequency of 11.518 Hz and oscillation frequency of 10 Hz; moment excitation and eight exciters are being used.

Table 3
Vibration mode 2 response due to different combinations of f_1 and f_2

f_1 (Hz)	f_2 (Hz)	R.m.s. (m) over 25 s
10.51	1	0.0076
19.51	10	0.0085
29.51	20	0.0085

Table 4
Vibration mode 1 response due to number of shakers and position

Number of exciters	Position of throw center along beam (m)	Radius/length (apparent radius of throw)	R.m.s. response over 25 s m	Peak response at ~25 s m
2	0.43	0.48	0.44	0.67
4	0.43	0.48	0.35	0.52
8	0.43	0.48	0.34	0.47

For this comparison the first and last exciters were located at the same positions on the beam, i.e., 120 and 750 mm from the supported end, Thus, the center of throw and radius of throw are the same for all three cases. The largest response is given by the use of two exciters only and is associated with the moment action of an ffs; the largest moment arm between exciters for the three cases examined is for the two-force exciter case. The forces spend 50% of their time acting at either position and combined with the largest moment arm gives the largest response.

The generalized force provided by a classical ffs to excite a cantilever beam when the center of throw is located at 0.43 m is given by $Q_1 = 1.25\,Pr/l$ [3], and the maximum value of the generalized co-ordinate is given by $q_1 = P/(2K\xi)$. Substituting data from Table 1 into these relations along with a force amplitude of 1 N gives a response of 0.95 m. This result is 1.42 that of simulated exciter array data given in Table 4. Thus, the exciter array does not perform as well as the classical ffs, however, it is impossible to obtain r/l equal to 0.47 in actual operation, and a more likely value may be 0.01. Thus, the use of the array of exciters to frequency shift generalized force is a positive development.

3. Experimental verification

A laboratory rig was built to experimentally verify that the travelling impulse technique could be used to identify and excite a mode of vibration of a cantilever beam. The rig is shown in Fig. 8; the rig consists of a cantilever beam, properties of which are given in Table 1, eight PZT exciters attached to the beam between 120 and 750 mm from the fixed end, an eight channel car distributor with cabling attached to the individual PZTs and an electric motor that drives the car distributor

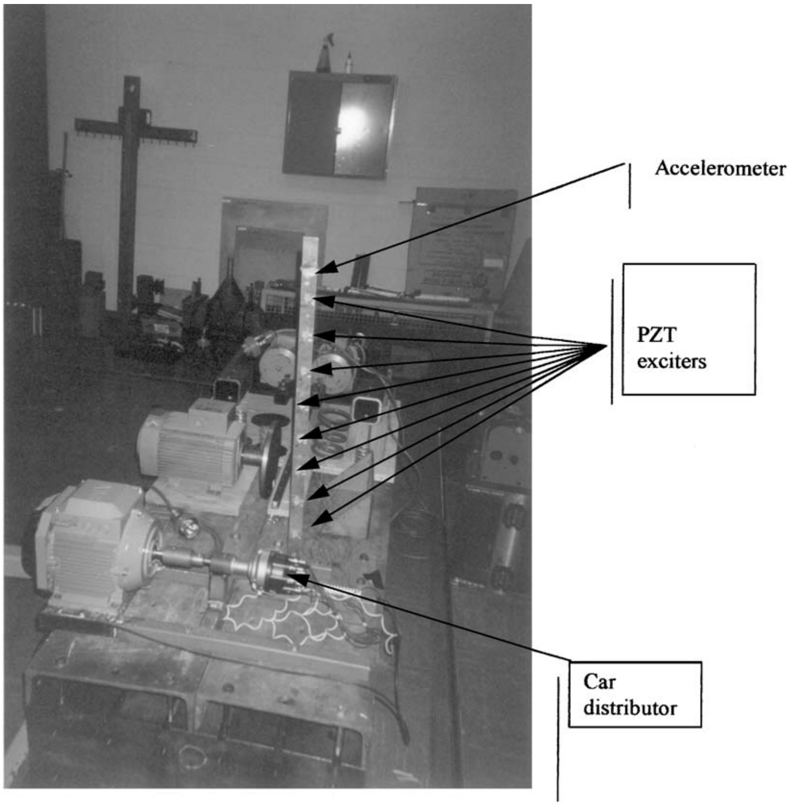

Fig. 8. Photograph of laboratory rig used to demonstrate frequency shifting and mode isolation of a cantilever beam.

(please note that an eight channel IO card was not available for this test). A voltage signal from a sine wave generator was fed into the center pin of the distributor at the frequency f_1 and the signal was distributed to the PZTs in the following sequence (PZT 1 is closest to the fixed end of the beam and PZT 8 was the furthest from the fixed end): 1–3–5–7–8–6–4–2–1 and repeat etc. This sequence simulated a force load that moves forward and backward along the beam. The motor ran the distributor at frequency f_2 and thus a difference frequency $f_1 - f_2$ was generated; the difference frequency was chosen to equal the measured natural frequencies. Thus, a travelling load using impulses was experimentally simulated. A PCB 353A12 accelerometer was attached to the beam to monitor acceleration; a PCB E09 ICP power supply powered the accelerometer and an AND 3525 dual channel spectrum measured the acceleration.

The measured first mode natural frequency was 1.35 Hz, and the measured second mode natural frequency was 9.45 Hz, both with accelerometer attached to the beam. For identification of both modes the motor was kept at a 10 Hz running speed and the sine wave signal to the PZTs was 11.35 Hz for mode 1identification and 19.40 Hz for mode 2 identification. The mode 1 results are graphically presented in Fig. 9 as a time history in the upper trace and as a frequency spectrum in the lower trace; the details are given in the figure caption. Only mode 1 is excited although some noise exists on the time history trace due to the low voltage level of the accelerometer signal. The

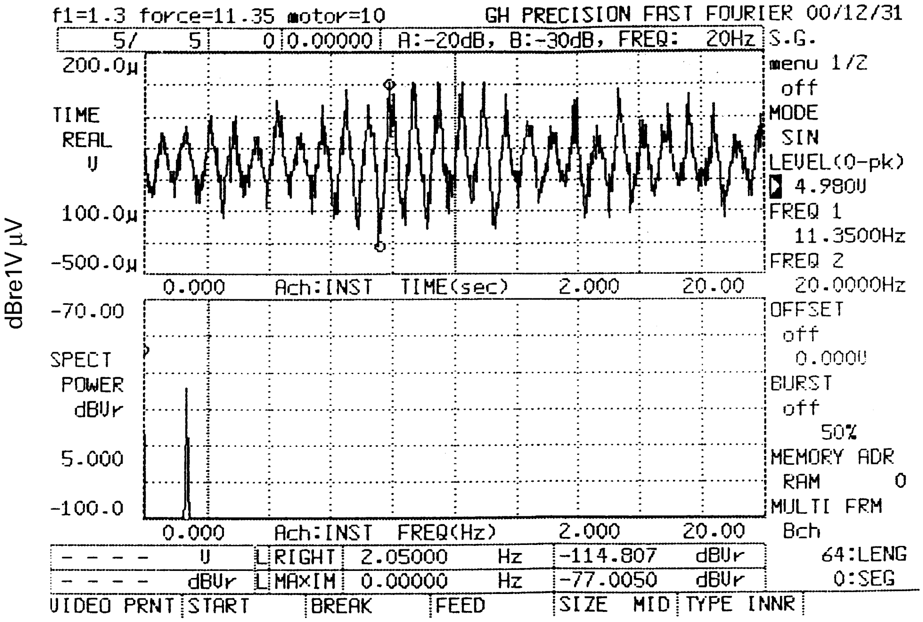

Fig. 9. Time history, top trace and acceleration vibration spectrum, bottom trace, from AND analyzer; cantilever beam mode 1 excited and identified. Time scale is 20 s with 2 s steps; frequency scale is 20 Hz with 2 Hz steps. Time history vertical scale is from −500 to 200 µV; spectrum vertical scale is from −70 to −100 dB re 1 V. Forcing frequency is 11.35 Hz, oscillation frequency is 10 Hz and natural frequency is 1.35 Hz. Note that second mode is not excited although high-frequency noise exists in the time history.

result shown in Fig. 10 clearly shows the second mode and the first mode amplitude about 10 dB lower than the second mode amplitude. The first mode was easier to identify using eight PZTs and the moving impulses than the second mode.

4. Conclusions

Force frequency shifting has been achieved by the use of an array of force exciters positioned along the length of a beam by experiment and demonstrated by simulation. Also, the generalized force amplitude at the lower structural frequency for a constant amplitude high-frequency force is independent of the frequencies used to accomplish the frequency shift. The use of a spatial array of exciters overcomes the difficulty in using a classical ffs shaker that has a low value of r/l. This investigation has shown that an array of force exciters can be employed to excite and identify low-frequency modes of vibration for modal analysis.

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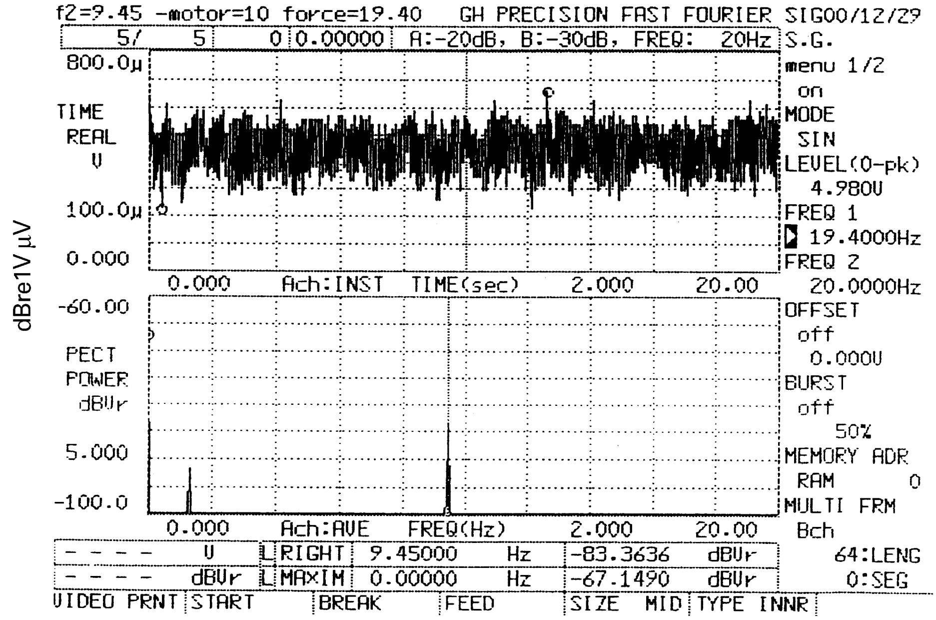

Fig. 10. Time history, top trace, and acceleration vibration spectrum, bottom trace, from AND analyzer; cantilever beam mode 2 excited and identified. Time scale is 20 s with 2 s steps; frequency scale is 20 Hz with 2 Hz steps. Time history vertical scale is from −500 to 200 μV; spectrum vertical scale is from −60 to −100 dB re 1 V. Forcing frequency is 19.40 Hz, oscillation frequency is 10 Hz and natural frequency is 9.40 Hz. Mode 1 is excited but is 10 dB down from mode 2 response.

References

[1] L.L. Koss, Fluctuating moment shaker for frequency shifting and structural excitation, Proceedings of the Third International Conference on Motion and Vibration Control, The Japanese Society of Mechanical Engineers, Chiba, Japan, 1996, pp. 258–261.

[2] L.L. Koss, X. Wang, Bridge and beam response to harmonic spatial and time loads, Fifteenth International Modal Analysis Conference, Society for Experimental Mechanics, Orlando, FL, USA, 1997, pp. 901–904.

[3] L.L. Koss, Force frequency shifting for structural excitation, Journal of Sound and Vibration 219 (2) (1999) 223–237.

[4] M.W. Trethewey, L.L. Koss, Application of a low frequency moment shaker for vibration testing, Eighteenth International Modal Analysis Conference, Society for Experimental Mechanics, San Antonio, TX, USA, 2000, Paper 227.

[5] M.W. Trethewey, L.L. Koss, Characteristics of force frequency shifting for low frequency excitation, Nineteenth International Modal Analysis Conference, Society for Experimental Mechanics, Orlando, FL, USA, 2001, Paper 173.

[6] L.L. Koss, M.W. Trethewey, Application of frequency shifting shaker to low frequency modal excitation, International Conference on Applications of Modal Analysis, Surfers Paradise, Queensland, Australia, 1999, Paper 2.

[7] L.L. Koss, Excitation of vibration in large structures, The Shock and Vibration Digest 33 (6) (2001) 473–478.

[8] J.W. Pate, Jr., Dynamic testing of a highway bridge, Fifteenth International Modal Analysis Conference, Society for Experimental Mechanics, Orlando, FL, USA, 1997, pp. 2028–2037.

[9] J. Li, B. Samali, Control of a five story building model under benchmark earthquake using SMA actuators, Proceedings of the Fifth International Conference on Motion and Vibration Control, Sydney Australia, The Japanese Society of Mechanical Engineers, 2000, pp. 173–179.
[10] N. Haritos, Experimental modal testing of reinforced concrete bridges, Fifth Secde Conference on European Seismic Design, 1995, pp. 93–100.
[11] L.L. Koss, T. Gal, W. Chiu, Force frequency shifting analysis using a finite element routine, Proceedings of the Fifth International Conference on Motion and Vibration Control, Sydney, Australia, The Japanese Society of Mechanical Engineers, 2000, pp. 609–612.
[12] W.T. Thomson, Vibration Theory and Applications, Prentice-Hall, Englewood Cliffs, NJ, 1965, p. 300.